# AIRBLAST FROM NUCLEAR BURSTS—ANALYTIC APPROXIMATIONS 

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Figure 1. Shock-front parameters (in shock-front frame of reference).

$$
\begin{align*}
\rho_{0} U_{s} & =\rho_{s}\left(U_{s}-u_{s}\right) \\
P_{s}-P_{0} & =\rho_{0} u_{s} U_{s} \tag{2}
\end{align*}
$$

(mass),
(momentum),
$\frac{1}{2} U_{s}^{2}+\frac{\gamma_{0}}{\left(\gamma_{0}-1\right)} \frac{P_{0}}{\rho_{0}}=\frac{1}{2}\left(U_{s}-u_{s}\right)^{2}+\frac{\gamma_{s}}{\left(\gamma_{s}-1\right)} \frac{P_{s}}{\rho_{s}} \quad$ (energy).

Equation (3) may also be written as

$$
\begin{equation*}
E_{S}-E_{0}=\frac{P_{S}}{\left(\gamma_{S}-1\right) \rho_{S}}-\frac{P_{0}}{\left(\gamma_{0}-1\right) \rho_{0}}=\frac{\left(P_{S}+P_{0}\right)}{2}\left(\frac{1}{\rho_{0}}-\frac{1}{\rho_{S}}\right) \tag{4}
\end{equation*}
$$

where $E_{S}=$ shocked specific internal energy,
$E_{O}=$ ambient specific internal energy.

Implicit in Eqs. (3) and (4) is the ideal gas relation for specific internal energy $E$ :

$$
E=P /[\rho(\gamma-1)],
$$

and (or)

$$
\begin{equation*}
P / \rho^{\gamma}=\text { constant } \text {, } \tag{5}
\end{equation*}
$$

for adiabatic flows, in which $\gamma$ is the specific heat ratio, $\gamma \equiv C_{p} / C_{V}$, where $C_{p}$ is the specific heat at constant pressure, and $C_{V}$ is the specific heat at constant volume, and where $P \equiv$ pressure and $\rho \equiv$ density.

SHOCK DENSITY.
Solving Eqs. (1), (2), and (3) for the shock density as a function of the peak pressure $P_{S}$ leads to

$$
\frac{\rho_{s}}{\rho_{0}}=\frac{\left(\frac{\gamma_{s}+1}{\gamma_{s}-1}\right)\left(\begin{array}{l}
P_{s}  \tag{6}\\
P_{0} \\
0
\end{array}\right)+1}{\left(\frac{\gamma_{0}+1}{\gamma_{0}-1}\right)+\frac{P_{s}}{P_{0}}}
$$

Expressed in terms of shock overpressure ( $\left.\Delta P_{S} \equiv P_{S}-P_{0}\right)$, the relation becomes

$$
\begin{equation*}
\frac{\rho_{s}}{\rho_{0}}=\frac{\left(\frac{\gamma_{s}+1}{\gamma_{s}-1}\right) \frac{\Delta P_{s}}{P_{0}}+\frac{2 \gamma_{s}}{\gamma_{s}-1}}{\frac{2 \gamma_{0}}{\gamma_{0}-1}+\frac{\Delta P_{s}}{P_{0}}} . \tag{7}
\end{equation*}
$$



Figure 2. Shock density versus shock overpressure in standard, sea-level air ( $\rho_{0} \approx 1.293 \mathrm{~kg} / \mathrm{m}^{3}$ ).


Figure 3. Specific heat ratio ( $\gamma \equiv C_{p} / C_{v}$ ) for shocks in standard, sea-level air versus shock pressure.

Figure 6. Shock dynamic pressure relative to shock overpressure


NORMAL REFLECTION OF SHOCKS.
When a shock wave strikes a rigid plane head on (as illustrated in Fig. 8), the reflected shock conditions can be derived from conservation considerations similar to those expressed in Eqs. (1), (2), and (4), for mass, momentum, and energy across the reflected shock:

$$
\begin{array}{cc}
\rho_{R} U_{R}=\left(u_{S}+U_{R}\right) \rho_{S} & \text { (mass), } \\
P_{R}-P_{S}=\left(u_{S}+U_{R}\right)^{2} \rho_{S}-U_{R}^{2} \rho_{R}=u_{S} U_{R} \rho_{R} & \text { (momentum), } \\
\frac{P_{R}}{\rho_{R}\left(\gamma_{R}-1\right)}-\frac{P_{S}}{\rho_{S}\left(\gamma_{S}-1\right)}=\frac{P_{R}+P_{S}}{2}\left(\frac{1}{\rho_{S}}-\frac{1}{\rho_{R}}\right) & \text { (energy), } \tag{28}
\end{array}
$$

in which subscripts R refer to conditions after reflection and subscripts $\underline{s}^{\text {apply }}$ to shock values prior to reflection.


Figure 8. Reflected shock conditions.

Equations (26), (27), and (28) together with the usual Hugoniot relations [Eqs. (1), (2), and (4)], lead to a quadratic in the reflection factor $R F \equiv\left(\Delta P_{R} / \Delta P_{S}\right)$ :

$$
\begin{align*}
\left(\frac{\Delta P_{R}}{\Delta P_{S}}\right)^{2} & +\left(\frac{\Delta P_{R}}{\Delta P_{S}}\right)\left\{\left(\frac{\gamma_{S}-\gamma_{R}}{\gamma_{S}-1} \frac{P_{0}}{\Delta P_{S}}\right)-\left(\frac{\gamma_{S}+\gamma_{R}-2}{\gamma_{S}-1}\right)-\left(\gamma_{R}+1\right)\right. \\
& \left.\times \frac{\left[\left(\frac{\gamma_{0}-1}{\gamma_{S}-1}\right) \Delta P_{S}-\left(\frac{\gamma_{0}-\gamma_{S}}{\gamma_{S}-1}\right) P_{0}\right]}{\left[2 \gamma_{0} P_{0}+\left(\gamma_{0}-1\right) \Delta P_{S}\right]}\right\}-\left(\frac{\gamma_{S}-\gamma_{R}}{\gamma_{S}-1}\right) \frac{P_{0}}{\Delta P_{S}}+\left(\frac{\gamma_{R}-1}{\gamma_{S}-1}\right) \\
& -\left[\left(\frac{\gamma_{0}-1}{\gamma_{S}-1}\right)-\frac{P_{0}}{\Delta P_{S}}\left(\frac{\gamma_{0}-\gamma_{S}}{\gamma_{S}-1}\right)\right] \frac{\left[2 \gamma_{R} P_{0}+\left(\gamma_{R}-1\right) \Delta P_{S}\right]}{\left[2 \gamma_{0} P_{0}+\left(\gamma_{0}-1\right) \Delta P_{S}\right]}=0 . \tag{29}
\end{align*}
$$

For an ideal gas (where $\gamma_{R}=\gamma_{S}=\gamma_{O}=\gamma$ ), that expression simplifies to:

$$
\begin{equation*}
R F \equiv \frac{\Delta P_{R}}{\Delta P_{S}}=\frac{2+\left(\frac{3 \gamma-1}{2 \gamma}\right)\left(\frac{\Delta P_{S}}{P_{0}}\right)}{1+\left(\frac{\gamma-1}{2 \gamma}\right)\left(\frac{\Delta P_{S}}{P_{0}}\right)}=2+(\gamma+1) \frac{Q_{S}}{\Delta P_{S}}, \tag{30}
\end{equation*}
$$

in which $\Delta \mathrm{P}_{\mathrm{R}}$ is the reflected peak overpressure from a normally incident shock of peak overpressure $\Delta P_{S}$ in an ambient atmosphere of pressure $P_{0}$, and $Q_{S}$ is given by Eqs. (16), (17), or (18). Equation (18), though, is an expression for $Q_{S} / \Delta P_{S}$ for a shock in real (sea-level) air, and is therefore inconsistent with the ideal gas assumption of Eq. (30).

For air at sea level, a better approximation to the reflection factor RF is given by the formula

$$
\begin{align*}
R F= & 2+\frac{2.655 \pi}{1+0.1728 \pi+0.001921 \pi^{2}} \\
& +\frac{0.004218+48.34 \pi+6.856 \pi^{2}}{1+7.997 \pi+3.844 \pi^{2}}, \tag{31}
\end{align*}
$$

where $\pi=\Delta P_{S} / 1000,\left(\Delta P_{S}\right.$ in pounds per square inch).

For a strong shock ( $\Delta P_{S} \gg P_{0}$ ), the expression in Eq. (29)
simplifies to

$$
\begin{equation*}
R F=1+\frac{2 \gamma_{R}}{\gamma_{S}-1} . \tag{32}
\end{equation*}
$$

In Fig. 9, the reflection factor predicted by the approximation in Eq. (31) is compared with a more exact solution from Eq. (29) using the equation of state for air given in Sec. 5 [Eqs. (69) through (74)].

TABLE OF SHOCK VARIABLES.
The shock parameters treated in this section (and plotted in Figs. 2 through 7, and 9) are listed for a range of shock overpressures in Table 1, for standard, sea-level air. The specific heat ratio $\gamma_{S}$ is from Eq. (9), the shock density ratio $n_{S}$ from Eqs. (7) and (9), the shock velocity $U_{S}$ from Eqs. (9) and (10), the peak particle velocity $u_{S}$ from Eqs. (9) and (12), and the peak dynamic pressure $Q_{S}$ from Eqs. (9) and (16). The ratio of the gas constants (thermal) $v$ is given by Eq. (25); the shock temperature rise $\Delta \theta_{S}$ is provided by combining Eqs. (9), (20), and (25), and the normal reflection factor RF is arrived at from Eq. (29) and the equation-of-state for air, from Sec. 5, Eqs. (69) through (74).


Table 1. Shock variables.

| $\begin{aligned} & \Delta P_{S} \\ & (\mathrm{psi}) \end{aligned}$ | $\gamma_{s}-1$ | $\begin{gathered} \eta_{S} \\ \left(\rho_{S} / \rho_{0}\right) \end{gathered}$ | $\begin{gathered} U_{s} \\ (\mathrm{kft} / \mathrm{s}) \end{gathered}$ | $\begin{gathered} \mathrm{u}_{\mathrm{s}} \\ (\mathrm{kft} / \mathrm{s}) \end{gathered}$ | $\mathrm{Q}_{\mathrm{S}}$ (psi) | $\nu_{S}$ | $\begin{aligned} & \Delta \theta_{\mathrm{S}} \\ & \left({ }^{\circ} \mathrm{C}\right) \end{aligned}$ | $\begin{gathered} R F \\ \left(\Delta P_{R} / \Delta P_{S}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 1 | . 400 | 1.005 | 1.090 | . 00527 | . 000242 | 1.00 | 0.5 | 2.009 |
| . 15 | . 4 | 1.007 | 1.092 | . 00789 | . 000545 | 1.00 | 0.8 | 2.012 |
| . 2 | . 4 | 1.010 | 1.094 | . 01051 | . 00097 | 1.00 | 1.1 | 2.014 |
| . 3 | . 4 | 1.015 | 1.097 | . 01571 | . 00218 | 1.00 | 1.6 | 2.019 |
| . 4 | . 4 | 1.019 | 1.100 | . 02089 | . 00387 | 1.00 | 2 | 2.025 |
| . 6 | . 4 | 1.029 | 1.106 | . 0312 | . 00869 | 1.00 | 3 | 2.035 |
| . 8 | . 4 | 1.039 | 1.112 | . 0413 | . 01541 | 1.00 | 4 | 2.045 |
| 1 | . 4 | 1.048 | 1.118 | . 0514 | . 0261 | 1.00 | 5 | 2.055 |
| 1.5 | . 4 | 1.072 | 1.134 | . 0760 | . 0539 | 1.00 | 8 | 2.080 |
| 2 | . 4 | 1.095 | 1.149 | . 1000 | . 0953 | 1.00 | 10 | 2.105 |
| 3 | . 4 | 1.146 | 1.178 | . 1462 | . 2125 | 1.00 | 15 | 2.154 |
| 4 | . 4 | 1.187 | 1.207 | . 1903 | . 3742 | 1.00 | 20 | 2.202 |
| 6 | . 4 | 1.275 | 1.263 | . 2728 | . 8264 | 1.00 | 28 | 2.297 |
| 7 | . 4 | 1.318 | 1.290 | . 3116 | 1.1146 | 1.00 | 33 | 2.343 |
| 10 | . 4 | 1.443 | 1.368 | . 4200 | 2.214 | 1.00 | - 45 | 2.478 |
| 15 | . 4 | 1.636 | 1.489 | . 5788 | 4.771 | 1.00 | 54 | 2.692 |
| 20 | . 4 | 1.814 | 1.600 | . 7179 | 8.137 | 1.00 | 82 | 2.891 |
| 30 | . 4 | 2.129 | 1.803 | . 9559 | 16.93 | 1.00 | 117 | 3.254 |
| 40 | . 4 | 2.400 | 1.985 | 1.158 | 27.99 | 1.00 | 150 | 3.575 |
| 50 | . 399 | 2.640 | 2.150 | 1.336 | 41.01 | 1.00 | 182 | 3.862 |
| 70 | . 397 | 3.063 | 2.468 | 1.663 | 71.50 | 1.00 | 244 | 4.351 |
| 100 | . 392 | 3.522 | 2.833 | 2.028 | 126.1 | 1.00 | 332 | 4.930 |
| 150 | . 385 | 4.092 | 3.378 | 2.351 | 232.9 | 1.00 | 475 | 5.629 |
| 200 | . 378 | 4.508 | 3.844 | 2.990 | 350.8 | 1.00 | 612 | 6.126 |
| 300 | . 368 | 5.063 | 4.637 | 3.718 | 609.4 | 1.00 | 882 | 6.795 |
| 450 | . 353 | 5.602 | 5.612 | 4.608 | 1035.5 | 1.00 | 1268 | 7.413 |
| 700 | . 336 | 6.207 | 6.927 | 5.807 | 1822 | 1.00 | 1866 | 8.043 |
| 1000 | . 312 | 6.831 | 8.207 | 7.002 | 2916 | 1.00 | 2486 | 8.559 |
| 1500 | . 279 | 7.728 | 9.953 | 8.660 | 5046 | 1.00 | 3359 | 9.215 |
| 2000 | . 252 | 8.569 | 11.409 | 10.076 | 7569 | 1.01 | 4045 | 9.758 |
| 3000 | . 237 | 9.175 | 13.91 | 12.39 | 12263 | 1.10 | 5273 | 10.656 |
| 4000 | . 228 | 9.565 | 16.02 | 14.34 | 17130 | 1.18 | 6313 | 11.375 |
| 6000 | . 207 | 10.51 | 19.52 | 17.66 | 28530 | 1.24 | 8288 | 12.430 |
| 8000 | . 191 | 11.35 | 22.45 | 20.47 | 41390 | 1.34 | 9530 | 13.132 |
| 10000 | . 180 | 11.99 | 25.04 | 22.94 | 54940 | 1.46 | 10360 | 13.604 |
| 15000 | . 172 | 12.56 | 30.60 | 28.16 | 86660 | 1.81 | 12030 | 14.20 |
| 20000 | . 179 | 12.12 | 35.39 | 32.46 | 111200 | 2.08 | 14450 | 14.35 |
| 30000 | . 207 | 10.65 | 43.61 | 39.51 | 144700 | 2.62 | 19750 | 14.13 |
| 40000 | . 210 | 10.50 | 50.39 | 45.59 | 190000 | 3.12 | 22400 | 13.66 |
| 60000 | . 218 | 10.16 | 61.83 | 59.74 | 274800 | 4.04 | 26900 | 12.64 |
| 80000 | . 234 | 9.537 | 71.65 | 64.13 | 341500 | 4.76 | 32500 | 11.72 |
| 100,000 | . 232 | 9.612 | 80.07 | 71.74 | 430600 | 5.19 | 37000 | 10.95 |
| 150,000 | . 209 | 10.56 | 97.35 | 88.32 | 717200 | 5.90 | 44500 | 9.520 |
| 200,000 | . 212 | 10.43 | 112.7 | 101.9 | 942900 | 6.74 | 52600 | 8.552 |
| 300,000 | . 235 | 8.841 | 139.4 | 123.5 | 1176000 | 8.73 | 72000 | 7.341 |
| 400,000 | . 2545 | 8.857 | 160.9 | 142.8 | 1571000 | 9.75 | 85800 | 6.619 |
| 600,000 | . 254 | 8.873 | 197.1 | 174.9 | 2362000 | 11.0 | 113700 | 5.800 |
| 300,000 | . 264 | 8.575 | 228.1 | 201.5 | 3030000 | 11.7 | 148000 | 5.346 |
| 1000,000 | . 285 | 8.017 | 256.2 | 224.2 | 3508000 | 12.1 | 192000 | 3.060 |
| 1500,000 | . 353 | 6.665 | 318.4 | 270.0 | 4249000 | 12.3 | 339000 | 4.659 |
| 2000,000 | . 400 | 6.000 | 371.3 | 309.4 | 5000000 | 12.4 | 500000 | 4.449 |
| 3000,000 | . 391 | 6:115 | 453.9 | 379.6 | 7672000 | 12.5 | 731000 | 4.234 |
| 4000,000 | . 352 | 6.682 | 519.8 | 441.9 | 11363000 | 12.7 | 877000 1089000 | 4.124 4.012 |
| 6000,000 | . 304 | 7.579 | 630.1 | 546.5 | 19740000 | 13.5 | 1089000 | 4.012 |
| 8000,000 | . 316 | 7.329 | 729.4 | 629.9 | 25320000 | 15.1 | 1340000 1760000 | 3.956 3.921 |
| 10000,000 | . 363 | 6.510 | 828.8 | 697.2 | 2755000 |  |  |  |



Figure 10. Peak overpressure versus shock radius for 1-KT free-air nuclear burst in standard, sea-level air.

Peak overpressure (psi)


Peak overpressure (psi)

Figure 12. Time of arrival versus peak overpressure for 1-KT free-air burst at sea level.



Figure 14. Time of arrival versus shock radius for 1-KT free-air burst.


POSITIVE OVERPRESSURE DURATION VERSUS RANGE.
As a function of scaled range ( 1 KT , free-air burst), the duration of positive overpressure can be approximated by

$$
\begin{equation*}
D_{p}^{+}=(m)\left[69.12+\frac{46.19}{\left(1+3,000,000 r^{7.217}\right)}+\frac{4043 r^{6.329}}{\left(1+37.16 r^{5.621}\right)}\right] \mathrm{ms}, \tag{46}
\end{equation*}
$$

where $r$ is scaled range in kilofeet per cube-root kiloton. Again, for a surface burst, $D_{p}^{+}$should be increased by a factor of $2^{1 / 3}$, i.e., the free-air form for twice the yield.
overpressure Impulse in positive phase versus peak overpressure (freeAIR AND SURFACE BURST).

Positive phase impulse $I_{p}^{+}$is defined by the integral

$$
\begin{equation*}
I_{p}^{+} \equiv \int_{T}^{T+D_{p}^{+}} \Delta P(t) d t=m \int_{\tau}^{\tau+D_{p}^{+} / m} \Delta P(\sigma) d \sigma \tag{47}
\end{equation*}
$$

A simple approximation to this impulse as a function of peak overpressure is

$$
\begin{equation*}
I_{p}^{+}=\frac{145 \Delta P_{s}^{1 / 2}}{\left(1+0.00385 \Delta P_{s}^{1 / 2}\right)} \mathrm{m} \quad \text { psi-ms. } \tag{48}
\end{equation*}
$$

This fit, when used with Eq. (33), leads to the values compared in Fig. 16 versus radius. The impulse approaches a constant at small ranges and decays approximately as the inverse of the range elsewhere. For a surface burst, that expression should be multiplied by 1.26 (i.e., by $2^{1 / 3}$ ), which leads to replacing the coefficient 145 by 183. This form is good to better than 10 percent for $2<\Delta P_{S}<100,000$ psi. Comparison between the approximation as a function of peak overpressure and the detailed numerical results [Brode, 1964] is made in Fig. 17. In that plot, it is evident that impulse increases

Range ( $\mathrm{ft} / \mathrm{KT}^{1 / 3}$ )
Figure 16. Fit compared to detailed calculation and l-KT standard: overpressure impulse in positive phase (l-KT, free-air burst).

approximately as the cube root of overpressure, but tends toward a constant at about a few thousand pounds per square inch.
overpressure versus time.
The following analytic expression is valid for overpressures less than about 15,000 psi. It is an approximate form, modified from earlier fits [Brode, 1970, 1978] for the overpressure in the positive phase as a function of time. In these fits, time is zero at the instant of burst:
$\Delta P=\Delta P_{S}(\sigma)\left\{0.417+0.583\left(\frac{\tau}{\sigma}\right)^{6}\left[\frac{40\left(\frac{\tau}{\sigma}\right)^{6}+\tau^{2}}{40+\tau^{2}}\right]\right\}\left(1-\frac{\sigma-\tau}{D_{p}^{+} / m}\right)$ psi,
where $\Delta P_{S}(\sigma)$ is found in Eq. (37) and oreplaces $\tau, \tau \equiv T / m, \sigma=t / m$, $T$ is the time of arrival (in milliseconds), $t$ is the time after time of arrival ( $t \geqq T$, i.e., $\sigma \geqq \tau$ ). Both $T$ and $t$ are measured from the instant of detonation. The scale factor $m \equiv w^{1 / 3}$ (in cube-root kilotons) and $D_{p}^{+}$is the duration of the positive phase [Eqs. (43) through (46)]. As before, for a surface burst, use $m=(2 W)^{1 / 3}$.

OVERPRESSURE VERSUS TIME (SURFACE BURST).
Alternatively, one can use the zero burst height from the more complex equations for height of burst and range [Eq. (63) and Speicher and Brode, 1980a,b, 1981, 1984a,b]. When zero burst height is inserted and expressions are simplified, the calculation for a surface burst becomes

$$
\begin{equation*}
\Delta P=\Delta P_{S}(\tau)\left[f\left(\frac{\tau}{\sigma}\right)^{g}+(1-f)\left(\frac{\tau}{\sigma}\right)^{h}\right]\left(1-\frac{\sigma-\tau}{D}\right) \text { psi, } \tag{50}
\end{equation*}
$$


Peak overpressure (psi)
Figure 18. Fit compared to calculation and $1-K T$ standard (scaled to 1-MT surface burst): dynamic pressure positive phase.

numerical (one-dimensional) calculations is a measure of the differences introduced by dissimilarities in boundary and initial conditions, equations of state and opacities, and by various treatments of radiation transport, thermal radiation losses, and accumulated numerical errors in detailed computer calculations.

DYNAMIC IMPULSE VERSUS PEAK OVERPRESSURE (FREE-AIR BURST). The dynamic impulse in the positive phase, defined as

$$
\begin{equation*}
I_{u}^{+} \equiv(1 / 2) \int_{T}^{T+D_{u}^{+}} \rho u^{2} d t \tag{53}
\end{equation*}
$$

can be approximated as

$$
\begin{equation*}
I_{u}^{+}=\frac{2.14 \Delta P_{s}^{1.637}(\mathrm{~m})}{\left(1+0.00434 \Delta P_{s}^{1.431}\right)} \quad \text { psi-ms. } \tag{54}
\end{equation*}
$$

That form is within 10 percent of the scaled values from the detailed calculations [Brode, 1959b, 1966] for $3<\Delta P_{S}<10,000$ psi. It is high by nearly 20 percent at $\Delta P_{S}=100,000$ psi. Figure 19 compares the dynamic impulse from the detailed calculations with that from this fit (Eq. 54).

DYNAMIC IMPULSE VERSUS SCALED RANGE.
A fit to the dynamic impulse versus range for the early calculations [Brode, 1959b] agrees to better than 10 percent for $0.0025 \leqq r \leqq$ $2 \mathrm{kft} / \mathrm{KT}^{1 / 3}$. The relation, when scaled to a $1-\mathrm{KT}$ free-air burst, is

$$
I_{u}^{+}=\left[\frac{18.8 r^{2}}{10^{-6}+0.06896 r^{3}+0.5963 r^{5.652}}+\frac{92.64}{(100 r)^{5}}\right.
$$

$$
\begin{equation*}
\left.+\frac{2935(r-0.00597)(0.01-r)\left(0.0003552-r^{4}\right)}{10^{-10}+0.003377 r^{2.5}+155.8 r^{8}}\right] m \quad \text { psi-ms, } \tag{55}
\end{equation*}
$$

with $r=s r / m$, sr in kilofeet, $m=W^{1 / 3}$, and $W$ in kilotons. This expression is illustrated in Fig. 20 and compared with the detailed calculation results to which it was fit. The fit is good to a few percent over the entire range. For a surface burst, $m=(2 W)^{1 / 3}$.

## DYNAMIC PRESSURE VERSUS TIME.

An older approximate analytic expression for dynamic pressure versus time covers the range $2 \leqq \Delta P_{X} \leqq 1000 \mathrm{psi}\left(0.1 \leqq \mathrm{Q}_{\mathrm{S}} \leqq 3000 \mathrm{psi}\right)$ [Brode, 1964]:

$$
\begin{equation*}
Q(t)=Q_{s}(1-\omega)^{2}[d \exp (-a \omega)+(1-d) \exp (-b \omega)] \quad \text { psi } \tag{56}
\end{equation*}
$$

in which $\omega \equiv(t-T) / D_{u}^{+}$,
T $\equiv$ time of arrival [see Eqs. (39) through (41)],
't $\equiv$ time,
$D_{u}^{+} \equiv$ duration of outward blast wind [see Eq. (52)],
$Q_{S}=$ peak dynamic pressure in pounds per square inch [see Eqs. (16) through (18)],

$$
\begin{aligned}
& d=\frac{1.06 \pi^{0.035}}{1+147 \pi^{3}}+\frac{2.13 \pi^{3}}{1+67.9 \pi^{3.5}}, \\
& a=0.38 \Delta P_{S}^{0.8605}=145 \pi^{0.8605}, \\
& b=5.4 \Delta P_{S}^{0.604} \approx 350 \pi^{0.604} .
\end{aligned}
$$

Equation (56) is valid for $1 \geqq \pi \geqq 0.002 \mathrm{ksi}\left(1000 \geqq \Delta \mathrm{P}_{\mathrm{x}} \geqq 2 \mathrm{psi}\right)$. These parameters are illustrated in Fig. 21 (scaled to 1-MT surface burst).

A relatively simple alternative surface burst formula for dynamic pressure versus time can be derived from the more complex fits to dynamic and overpressure versus HOB, ground range, and time [Speicher, 1983; Speicher and Brode, 1981; Brode, 1983]. That fit, when simplified for zero $H O B$, becomes essentially that of the shock or


Figure 20. Fit compared to calculation: scaled dynamic pressure impulse versus scaled range for $1-K T$ free-air burst.


Figure 21. Dynamic pressure versus peak overpressure and time (scaled for $1-M T$ surface burst).


Figure 41. High peak overpressures versus scaled burst height and scaled ground range (ideal surface).
 Scaled ground range ( $\mathrm{ft} / \mathrm{K}^{1 / 3}$ )

- Smiley, Tomayko, and Ruetenik [1982a-d]
Figure 110. Integration of Eq. (66) over positive phase compared to Kaman AviDyne calculations: dynamic impulse versus SHOB and scaled ground range.


For strong blast waves in the atmosphere, the real gas (nonideal gas) properties of air become important, and the ideal gas assumptions frequently prove inadequate. Air molecules enter a complex energy-density-pressure balance as they dissociate and ionize with rising pressure or temperature. Their behavior is expressed in thermodynamic terms from detailed calculations [Gilmore, 1955, 1959; Hilsenrath, Green, and Beckett, 1957]. The equation of state for air has been closely fit by the author [Brode and Parkin, 1963] and that fit has been used in detailed numerical calculations of the radiation flow and hydrodynamics of nuclear bursts [Brode, 1959a,b, 1966, 1969; Brode et al., $1 \neq 67$ ], as well as in the KA calculations [Smiley, Tomayko, and Ruetenik, 1982a-d].

CALORIC EQUATION OF STATE FOR AIR.
The usual ideal gas relation for specific internal energy can be written as

$$
\begin{equation*}
E=\frac{P}{\rho(\gamma-1)}, \tag{69}
\end{equation*}
$$

in which $E$ is the energy per unit mass, $P$ is the pressure, $\rho$ is the density, and $\gamma$ is the ratio of specific heats $\left(\gamma \equiv C_{p} / C_{V}\right)$. Defining $\mu=(\gamma+1) /(\gamma-1)$, one can rewrite Eq. (69) as

$$
\begin{equation*}
P=\frac{2 p E}{(\mu-1)} . \tag{70}
\end{equation*}
$$

Using a dimensionless variable defined as

$$
\begin{equation*}
\phi \equiv\left(\frac{P_{0}}{P}\right)\left(\frac{\rho}{\rho_{0}}\right)^{1.0553} \tag{71}
\end{equation*}
$$



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Figure 122. Comparison of thermal fractions as predicted by three formulae (Eqs. (101), (102), (103)) versus burst altitude for 10 MT .

Brode, H. L., Calculation of the Blast wave from a Spherical Charge of TNT, The Rand Corporation, Santa Monica, California, (subsequently published in Physics of Fluids, Vol. 2, March 1959a, pp. 217-229).
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